Maths for Computing
Tutorial 12

1. Let G be a bipartite graph with partite sets X and Y in which any vertex of X has degree at least as large as the degree of any vertex of Y. We also know that $\delta(G) \geq 1$. Prove that there exists a matching that covers X.

2. Let G be a k-regular bipartite graph for $k \geq 1$. Prove that G has a perfect matching.

3. A connected bipartite graph G has partite sets A and B, where $|A| = |B| = k \geq 2$. Prove that if every two vertices of A have distinct degrees in G, then G contains a perfect matching.

4. Prove that planar graphs are 6-colourable.

5. Prove that planar graphs are 5-colourable.

6. Prove that if a graph G has 11 or more vertices, then either G or $\overline{G}$ is not planar.